Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

# NOTE ON THE PRECEDING PAPER.

By Albert A. Bennett.

If we be given a one-valued real function $F(x)$, defined for all real values of $x$ including infinity, and if $F^n(x) \equiv x$, where the index $n$ refers to iteration, then we see that without imposing any further restriction, $F^{-1}(x)$, $[\equiv F^{n-1}(x)]$, is a one-valued real function defined for all real values of $x$ including infinity. If we look upon $x' = F(x)$ as a transformation of a set of real numbers into itself, it will be a one-one reciprocal transformation. We may take a circle and choose upon it an arbitrary point to represent infinity, and then represent all the finite real points, preserving order, by the remaining points of the circle: for example, one such method of representation is to let $x = \tan \frac{1}{2}\theta$, where $\theta$ is the central angle. The transformation $x' = F(x)$ then determines a one-one reciprocal transformation among the points of the circle. An obvious restriction is to require this transformation to be also continuous. *Thus, the problem of Mr. Ritt's paper may be viewed as that of discussing in terms of Analysis Situs, the different types of continuous one-one reciprocal transformations of period $n$ of a circle into itself.*

By transforming through a continuous, sense-preserving, one-one reciprocal transformation, the above problem of Analysis Situs becomes *normalized* into a problem of the periodic Euclidean transformations of period $n$ of a circle into itself, such as is discussed, for instance, in any treatise on linear fractional transformations in the complex plane. A mere statement of cases will here suffice:

1. Reflections (altering sense)—period two.
    (*a*) Leaving " $\infty$ " and another point fixed.
    (*b*) Leaving two points, but not " $\infty$ ", fixed.
2. Rotations (preserving sense)—period $n$.
    Leaving no point fixed—$(n - 1)$ cases; rotations of $1/n, 2/n, 3/n, \cdots, (n-1)/n$ circumferences.

In terms of *functions*, $y = F(x)$, these become:

1. Monotonic decreasing—period two.
    (*a*) No finite vertical or horizontal asymptote. Curve cuts line $y = x$ in one finite point.
    (*b*) A finite vertical and a finite horizontal asymptote. Curve cuts $y = x$ in two finite points.
2. Monotonic increasing—period $n$.

A finite horizontal and a finite vertical asymptote. Curve does not cut $y = x$, — $(n - 1)$ cases, corresponding to $f, f^2, f^3, \cdots, f^{n-1}$.

Princeton, N. J.,
September, 1915.